

## Regular Expressions & Languages

- ① A RL can be described using a RE
- ② RE is a form of notation consisting of the symbols of an alphabet  $\Sigma$  and operators  $+$ ,  $\cdot$  and  $*$ .
- $\uparrow$        $\uparrow$        $\uparrow$   
union    concatenation    closure

### Def of RE

Defined recursively as follows:  $\rightarrow$

- 1)  $\phi$  is a RE denoting an empty lang.
- 2)  $\epsilon$  is a RE and indicates the language containing an empty string
- 3)  $a$  is a RE which indicates the language containing only  $\{a\}$
- 4) If  $R$  is a RE which denotes a language  $L_R$ .

AND

$S$  is a RE that denotes the language  $L_S$ , then

(a)  $R + S$  is a RE  $\wedge L_R \cup L_S$  corresponding to

(b)  $R \cdot S$  is a RE  $\parallel L_R \cdot L_S$

(c)  $R^*$  is a RE  $\parallel L_R^*$

- 5) The expression obtained by applying any of the rules from 1 to 4 are RE's



$$(ab)^* + b(ab)^* + (ba)^* + a(ba)^* \quad (3)$$

Ex RE to accept a language consisting of strings of 0s and 1s with at most one pair of consecutive 0s

$$(1+01)^* \text{ combination.}$$

$$(1+01)^* 001^* \text{ at most, one pair of } 0s.$$

$$(1+01)^* 00(1+01)^* + (1+01)^* 0(1+01)^*$$

Ex RE for a language consisting of at least one a and at least one b.  $\Sigma = \{a, b, c\}$

$$(a+b+c)^*$$

at least one a and one b.

$$(1) \text{ ~~(a+b+c)^*~~ } c^* a (a+c)^* b$$

$$(2) c^* b (b+c)^* a$$

$$c^* a (a+c)^* b (a+b+c)^* + c^* b (b+c)^* a (a+b+c)^*$$

$$[c^* a (a+c)^* b + c^* b (b+c)^* a] (a+b+c)^*$$

Ex 4 RE for a language that 4  
accepts ~~even~~ strings of even length  
 $\Sigma = \{a, b\}$ .

aa ab ba bb — even strings  
 $(aa + ab + ba + bb)^*$

$$L(R) = \{ (aa + ab + ba + bb)^n \mid n \geq 0 \}$$

Ex 5 RE to accept string of odd length  
 $\Sigma = \{a, b\}$

$$(aa + ab + ba + bb)^* (a + b)$$

or

$$(a + b) (aa + ab + ba + bb)^*$$

Ex 6 RE for  $L(R) = \{ w \mid w \in \{0, 1\}^* \}$   
with at least 3 consecutive 0's

$(0+1)^*$  — string with 0's and 1's

$$(0+1)^* 000 (0+1)^*$$

$$L(R) = \left\{ (0+1)^m 000 (0+1)^n \mid \begin{array}{l} m \geq 0 \\ n \geq 0 \end{array} \right\}$$

Ex. ~~RE~~ RE to accept words with  $\textcircled{5}$   
2 or more letters but beginning  
and ending with the same letter  
 $\Sigma = \{a, b\}$

$(a+b)^*$   $\rightarrow$  consisting of a's and b's

$a(a+b)^*a + b(a+b)^*b$

$\uparrow$  beginning and ending  
with same letter

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Ex. RE to accept strings of a's and  
b's whose third symbol from  
right is a and fourth is b

$ba(a+b)(a+b)$  - 3rd and 4th

$(a+b)^*$  - string of a's and b's

$\uparrow$  can be concatenated at  
beginning or end.

$(a+b)^*ba(a+b)(a+b)(a+b)^*$